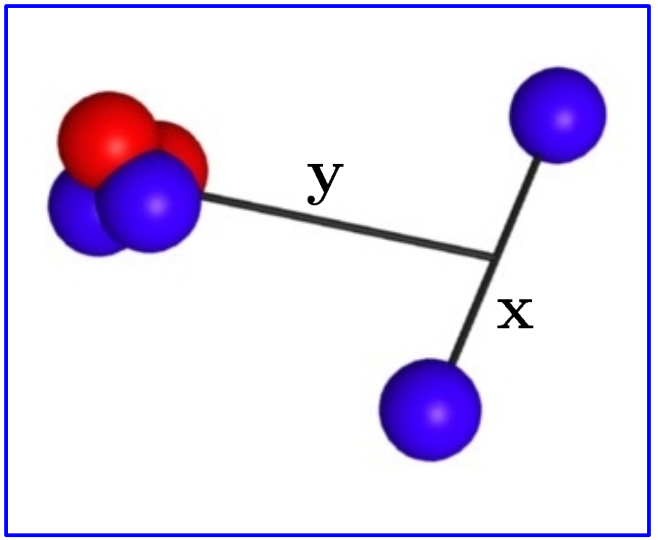
Continuum states of light nuclei by the 3-body microscopic cluster model



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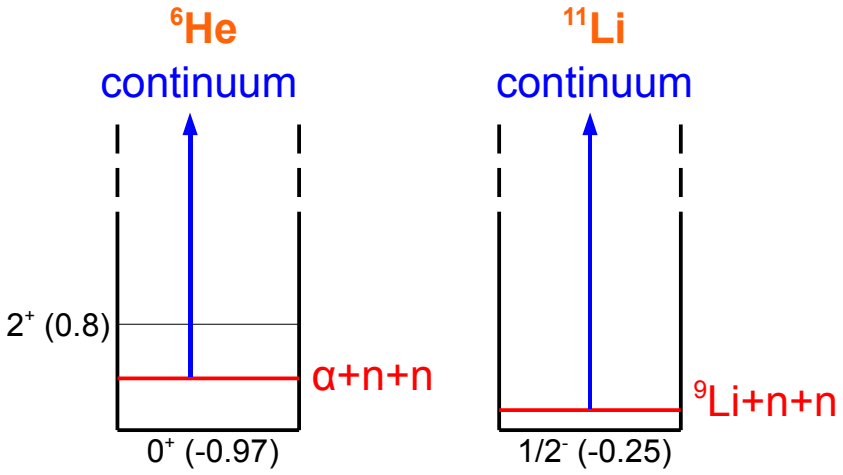
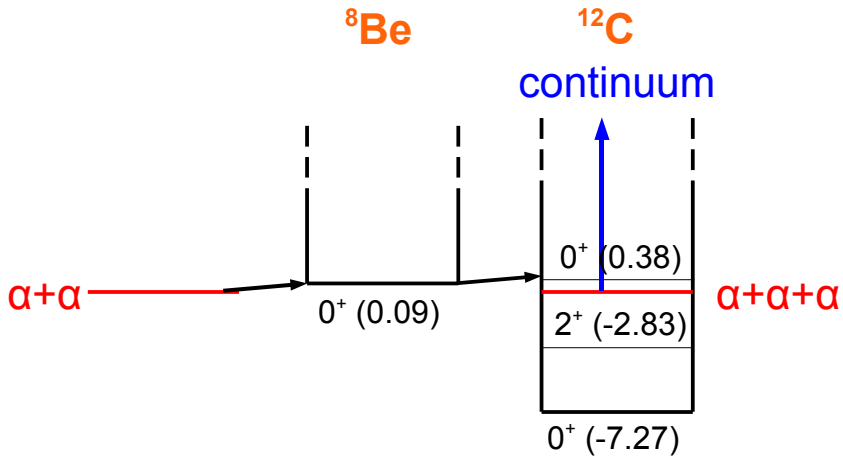
Introduction

- 80's : new radioactive beams (LLN, ISOLDE, TRIUMF, …) → study of exotic light nuclei

  [B. Jonson, Phys. Rep. 389 (2004) 1]

- Many examples of 3-cluster systems
  - Halo nuclei (core + 2n or 2p halo) : ^{6}He (α+n+n), ^{11}Li (^{9}Li+n+n)
  - Weakly bound nuclei : ^{9}Be (α+α+n)
  - Triple-α process (astrophysic interest) : ^{12}C (α+α+α)

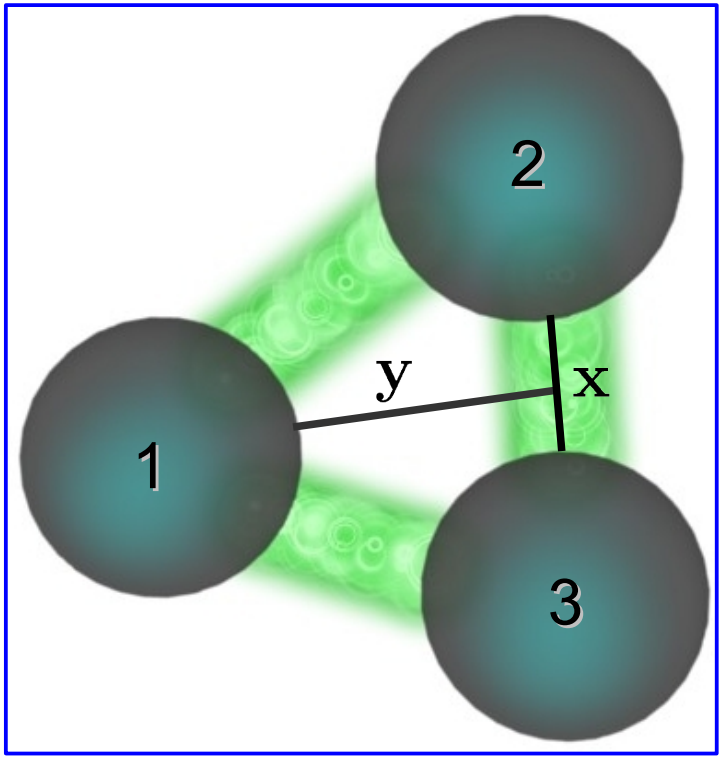




Very few bound states under 3-cluster threshold
→ continuum states are a wide field of investigation

Non-microscopic three-cluster Model

$$H = T_1 + T_2 + T_3 + V_{12} + V_{13} + V_{23}$$
$$\Psi^{JM\pi} = \Psi^{JM\pi}(\mathbf{y}, \mathbf{x})$$



- T_i : kinetic energy of cluster i (i=1,2,3)
- V_{ij} : potential between clusters i and j (i,j=1,2,3)
  - effective nucleus-nucleus potential adjusted from experimental data
  - (ex: $V_{\alpha\text{-}n} \rightarrow {}^{6}He$, $V_{^{9}Li\text{-}n} \rightarrow {}^{11}Li$, $V_{^{12}Be\text{-}n} \rightarrow {}^{14}Be$)
- Ψ : 3-cluster wave function
  - depends on 2 relative vector coordinates : **y** and **x**
- J^{π} : Total spin and parity → good quantum numbers
  - $J = L + S$
  - $L = \ell_y + \ell_x$
  - $\pi = \pi_1 \pi_2 \pi_3 \ (-)^{\ell_y + \ell_x}$ (π_i : parity of cluster i)

Non-microscopic three-cluster Model

- 3-body problem : no analytical solution → numerical calculations

- spherical coordinates : $(\mathbf{y},\mathbf{x}) = (y,x,\Omega_y,\Omega_x)$
  - 2 radial coordinates with range $[0,+\infty]$ → not well adapted for numerical calculations

- **hyperspherical coordinates** : $(\mathbf{y},\mathbf{x}) = (\rho,\alpha_\rho,\Omega_y,\Omega_x) = (\rho,\Omega^5)$ where $\begin{cases} \rho^2 = x^2+y^2 & (\rho : \text{hyperradius}) \\ \alpha_\rho = \arctan(y/x) & (\alpha_\rho : \text{hyperangle}) \end{cases}$
  - only 1 (hyper)radial coordinate with range $[0,+\infty]$ → makes calculations easier

Non-microscopic three-cluster Model

- Hamiltonian and 3-cluster wave function in the hyperspherical formalism (L and S fixed) :

$$H = -\left(\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{K^2(\Omega^5)}{\rho}\right) + V_{12} + V_{13} + V_{23}$$

$$\Psi^{JM\pi}(\rho,\Omega^5) = \sum_{K\ell_y\ell_x} \chi^{J\pi}_{K\ell_y\ell_x}(\rho)\ \mathcal{Y}^{JM}_{K\ell_y\ell_x}(\Omega^5)$$

$\mathcal{Y}^{JM}_{K\ell_y\ell_x}(\Omega^5)$ → (well-known)

- K² : 5-dimensional angular momentum operator
  - eigenfunctions : hyperspherical harmonics $\mathcal{Y}^{JM}_{K\ell_y\ell_x}(\Omega^5)$
  - eigenvalues : hypermomentum K

$$K^2\ \mathcal{Y}^{JM}_{K\ell_y\ell_x}(\Omega^5) = K(K+4)\ \mathcal{Y}^{JM}_{K\ell_y\ell_x}(\Omega^5)$$

- $\chi^{J\pi}_{K\ell_y\ell_x}$: hyperradial function → (to be determined)

Non-microscopic three-cluster Model

Theoretical studies

- Bound states → done ex : ^{6}He and ^{11}Li [M. V. Zhukov et. al., Phys. Rep. 231 (1993) 151]
- Continuum states → done ex : ^{6}He [I.J. Thompson et. al., Phys. Rev. C 61 (2000) 024318]

Drawbacks

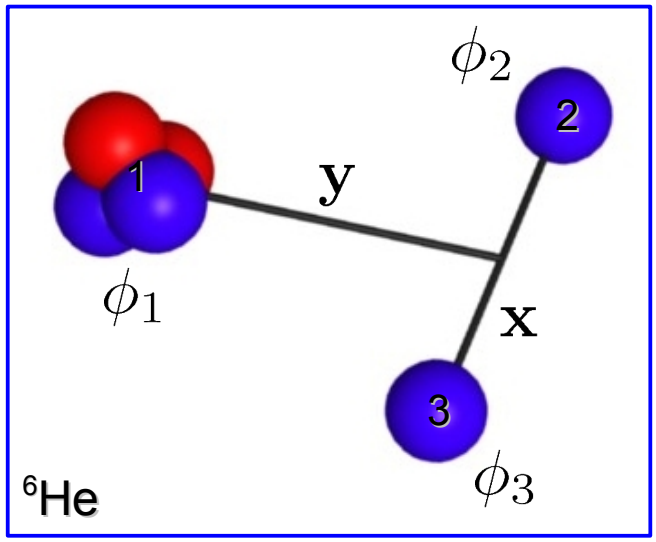
- Some nucleus-nucleus potentials are not defined precisely : $V_{^{9}\text{Li-n}}$, $V_{^{12}\text{Be-n}}$, ...
- Antisymmetrization is not treated exactly :
  - Pauli effects simulated by adapted potentials
  - internal structure of clusters neglected

Microscopic three-cluster Model

$$H = \sum_{i}^{A} T_i + \sum_{i \le j}^{A} V_{ij}$$

$$\Psi^{JM\pi} = \mathcal{A}\left[\phi_1\phi_2\phi_3 \sum_{K\ell_y\ell_x} \chi^{J\pi}_{K\ell_y\ell_x}(\rho)\ \mathcal{Y}^{JM}_{K\ell_y\ell_x}(\Omega^5)\right]$$

Antisymmetrizer



- A : number of nucleons
- T_i : kinetic energy of nucleon i (i=1,...,A)
- V_{ij} : potential between nucleons i and j (i,j=1,...,A)
  - effective nucleon-nucleon potential : Volkov, Minnesota potential (well determined)
  - predictive power
- Φ_k : internal cluster wave function (k=1,2,3)
- *A* : antisymmetrizer
  - Acts over all nucleons

Generator Coordinate Method (GCM)

$$\Psi^{JM\pi} = \mathcal{A}\left[\phi_1\phi_2\phi_3 \sum_{\lambda}^{N_\lambda} \chi_\lambda^{J\pi}(\rho)\ \mathcal{Y}_\lambda^{JM}(\Omega_5)\right]$$

$$= \sum_{\lambda}^{N_\lambda}\sum_{n}^{N_R} f_\lambda^{J\pi}(R_n)\ \Phi_\lambda^{JM\pi}(R_n)$$

In theory : infinite → In practice : finite (N_λ ,N_R)

- $f_\lambda(R)$: generator functions
  - coefficients of the variational method → functions depending on a discretized variational coordinate R
  - R → generator coordinate
- $\Phi_\lambda(R_n)$: projected Slater determinants based on H.O. Functions
  - constructed from a systematic analytical method (Mathematica, Mapple, ...)
  - basis size : N_λ x N_R
- $\Psi^{JM\pi}$ → Schrödinger equation $H\Psi^{JM\pi} = E^{J\pi}\Psi^{JM\pi}$ (numerical method) → bound states
  - ^{6}He and ^{6}Li → [S. Korennov, P. Descouvemont , Nucl. Phys. A 740 (2004) 249]

3-body scattering states

3-body scattering states ? : NOT investigated before

- Aim : calculate theoretical 3-body phase shifts
  - $\Psi^{JM\pi}$ into the whole space → including the asymptotic behavior :

$$\Psi^{JM\pi} \underset{\rho\to\infty}{\longrightarrow} \phi_1\phi_2\phi_3 \sum_\lambda \underbrace{\left(I_\lambda^{J\pi}\ \delta_{\lambda\lambda'} \ - \ U_{\lambda\lambda'}^{J\pi}\ O_{\lambda'}^{J\pi} \right)}_{\chi_{\lambda,as.}^{J\pi}} \mathcal{Y}_\lambda^{JM}$$

- Antisymmetrization can be neglected
- Only Coulomb interaction remains
  - I_λ , O_λ : incoming and outgoing Coulomb wave functions (analytical expression)
  - $U_{\lambda\lambda'}$: collision matrix → gives the 3-body phase shifts

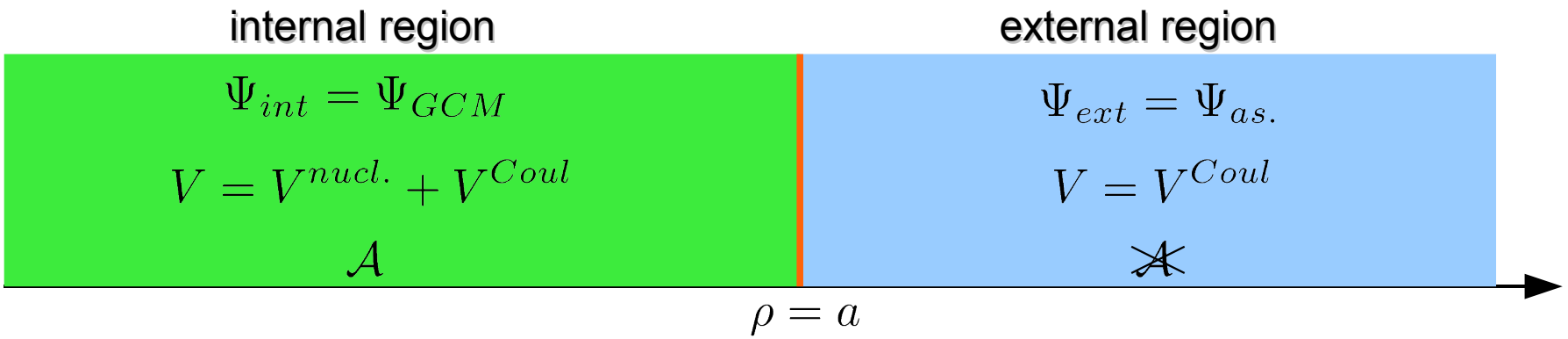
Problem

$$\chi_{\lambda,GCM}^{J\pi}(\rho) \underset{\rho\to\infty}{\not\longrightarrow} \chi_{\lambda,as.}^{J\pi}(\rho)$$

- the GCM failed to reproduce the asymptotic behavior of the hyperradial function

3-body scattering states

R-matrix method



3-cluster wave function calculated using the generator coordinate method

3-cluster wave function given by its asymptotic behavior (Coulomb wave functions I and O)

- a : channel radius
  - chosen so that antisymmetrisation and nuclear interaction can be neglected
- Matching at ρ = a → $\mathbf{R}^{J\pi}$ matrix
  - $\mathbf{R}^{J\pi}$ matrix → $\mathbf{U}^{J\pi}$ matrix — diagonalization → 3-body eigenphase shifts $\delta_\lambda^{J\pi}$
  - $\mathbf{U}^{J\pi}$ and $\delta_\lambda^{J\pi}$ should NOT depend on a

3-body scattering states

Calculation of the phase shifts → method in 3 steps :

Step 1 : construction of Slater determinants Φ → symbolic programming (Mathematica)

Step 2 : calculation of H (Hamiltonian) and N (norm) matrices → semi-analytical calculations

$$\mathbf{H}^{J\pi} = \langle \Phi^{JM\pi}_{\lambda}(R_n)|\ H\ |\Phi^{JM\pi}_{\lambda'}(R'_n)\rangle$$
$$\mathbf{N}^{J\pi} = \underbrace{\langle \Phi^{JM\pi}_{\lambda}(R_n)|\Phi^{JM\pi}_{\lambda'}(R'_n)\rangle}$$

7-dimensional integral for each (λ,R_n,λ',R'_n) !

→ 2 dimensions : numerical integration

→ 5 others : analytical integration

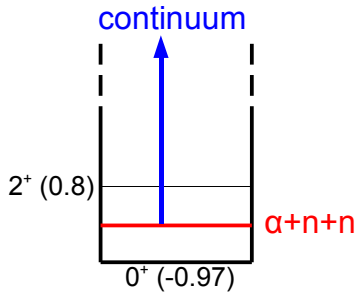
- time computing (analytical integration) << time computing (numerical integration)
- H and N : [N x N] matrices where $N = N_\lambda \times N_R$

Step 3 : calculation of the R matrix, the U matrix and the eigenphase shifts → numerical calculations

- R and U : [N_λ x N_λ] matrices
- diagonalization of U matrix → N_λ 3-body eigenphase shifts

^{6}He = α + n + n

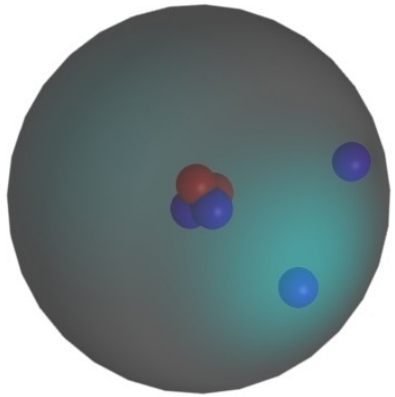
^{6}He states properties



State ($J\pi$)	Energy (MeV)	Γ (keV)
0^+	-0.975	
2^+	0.82	113± 20

[D. R. Tilley et. al., Nucl. Phys. A708 (2002) 3]

α core + 2n halo



Application on ^{6}He = α+n+n

- R_n = 1.25, …, 25 → N_R = 20

$N = N_\lambda \times N_R$

- Only S=0
- $K \leq 24$, $\ell_y \leq 4$, $\ell_x \leq 4$

J^π :

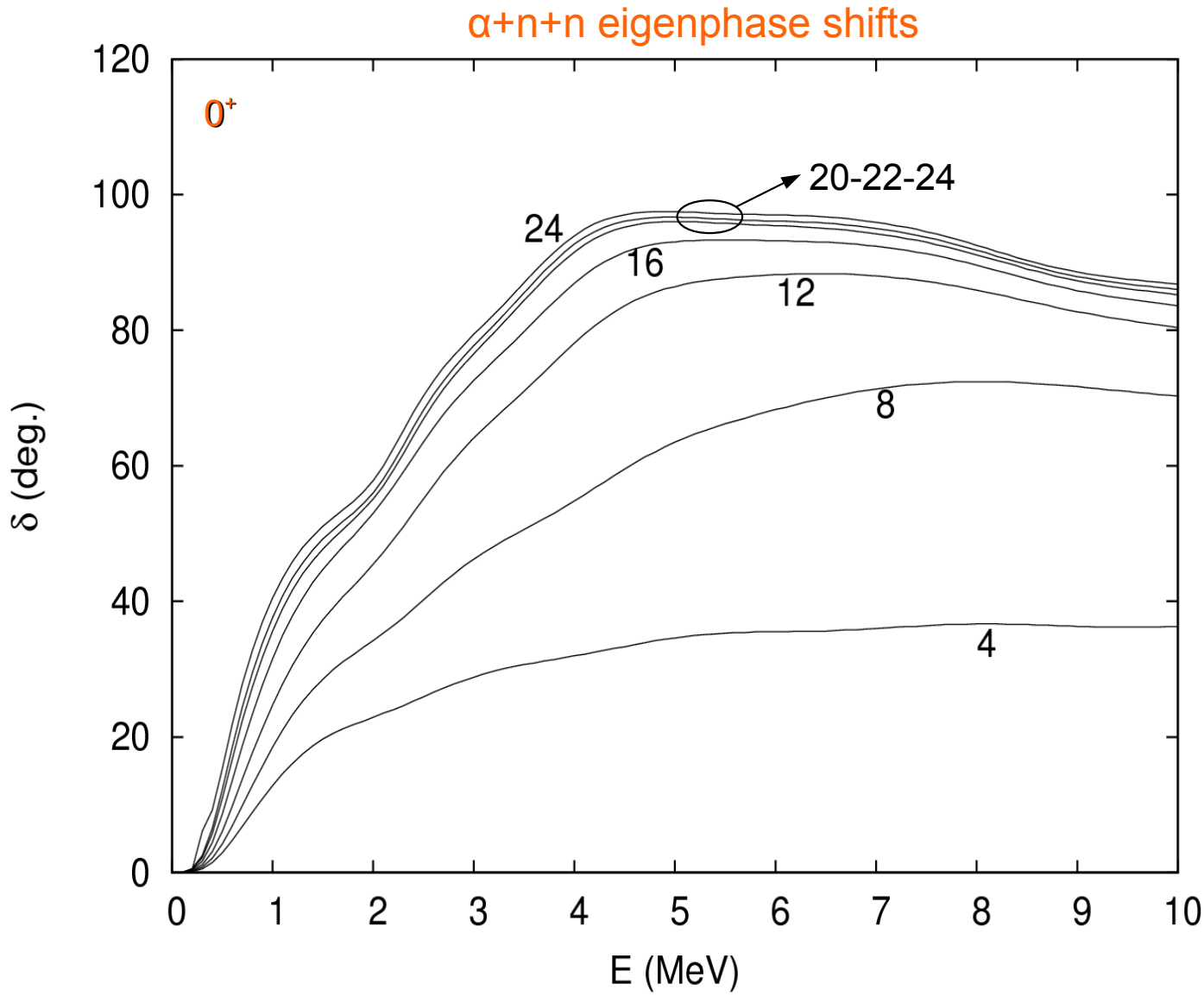
- $0^+ \rightarrow N_\lambda = 33$, $N = 660$
- $1^- \rightarrow N_\lambda = 42$, $N = 840$
- $2^+ \rightarrow N_\lambda = 64$, $N = 1280$

$\lambda=(K,\ell_y,\ell_x)$

^{6}He = α + n + n

Eigenphase shifts → convergence with K_{max}

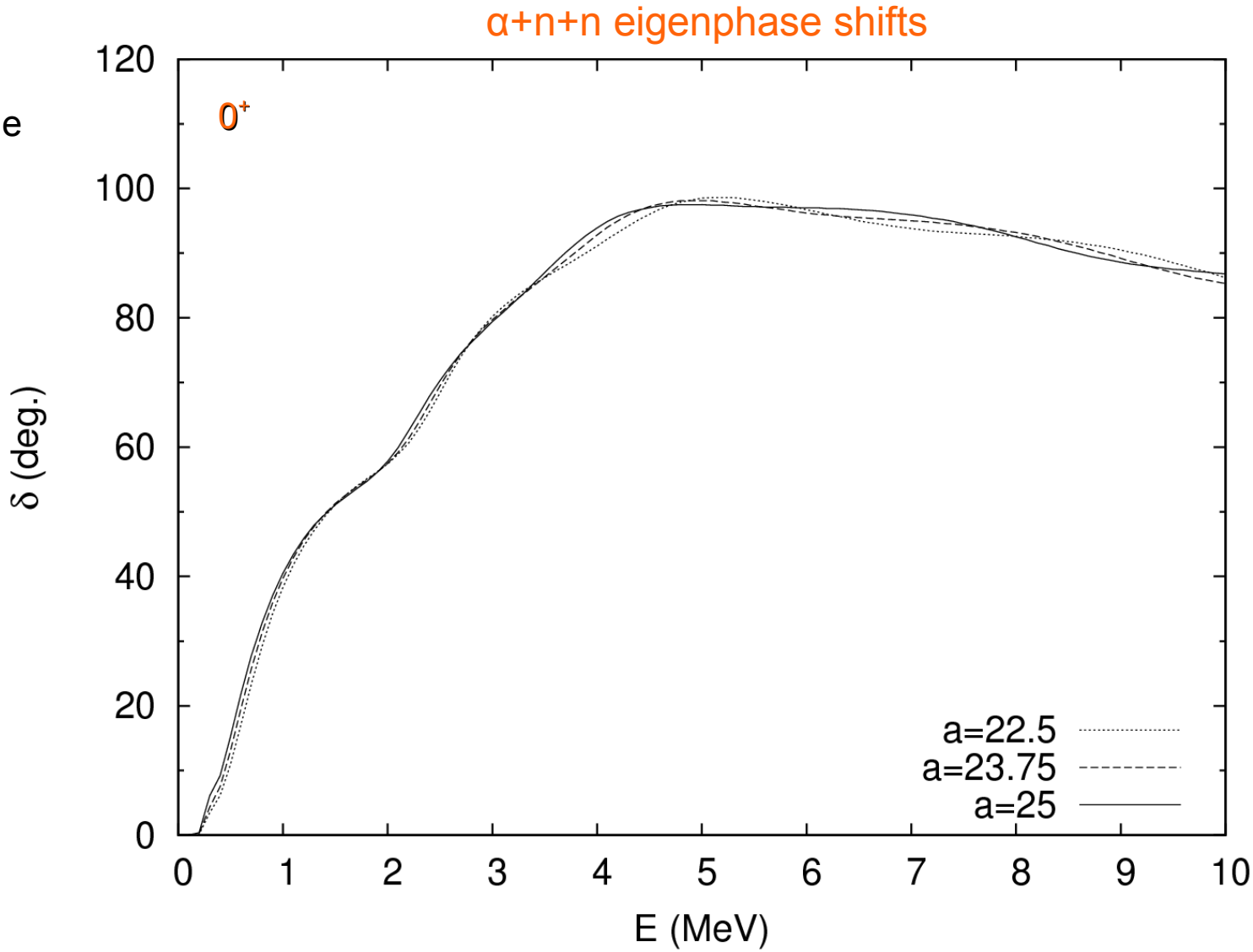
- Most important eigenphase shifts for different K_{max}
- No good convergence before K_{max}=24



^{6}He = α + n + n

Eigenphase shifts → Stability with the channel radius a

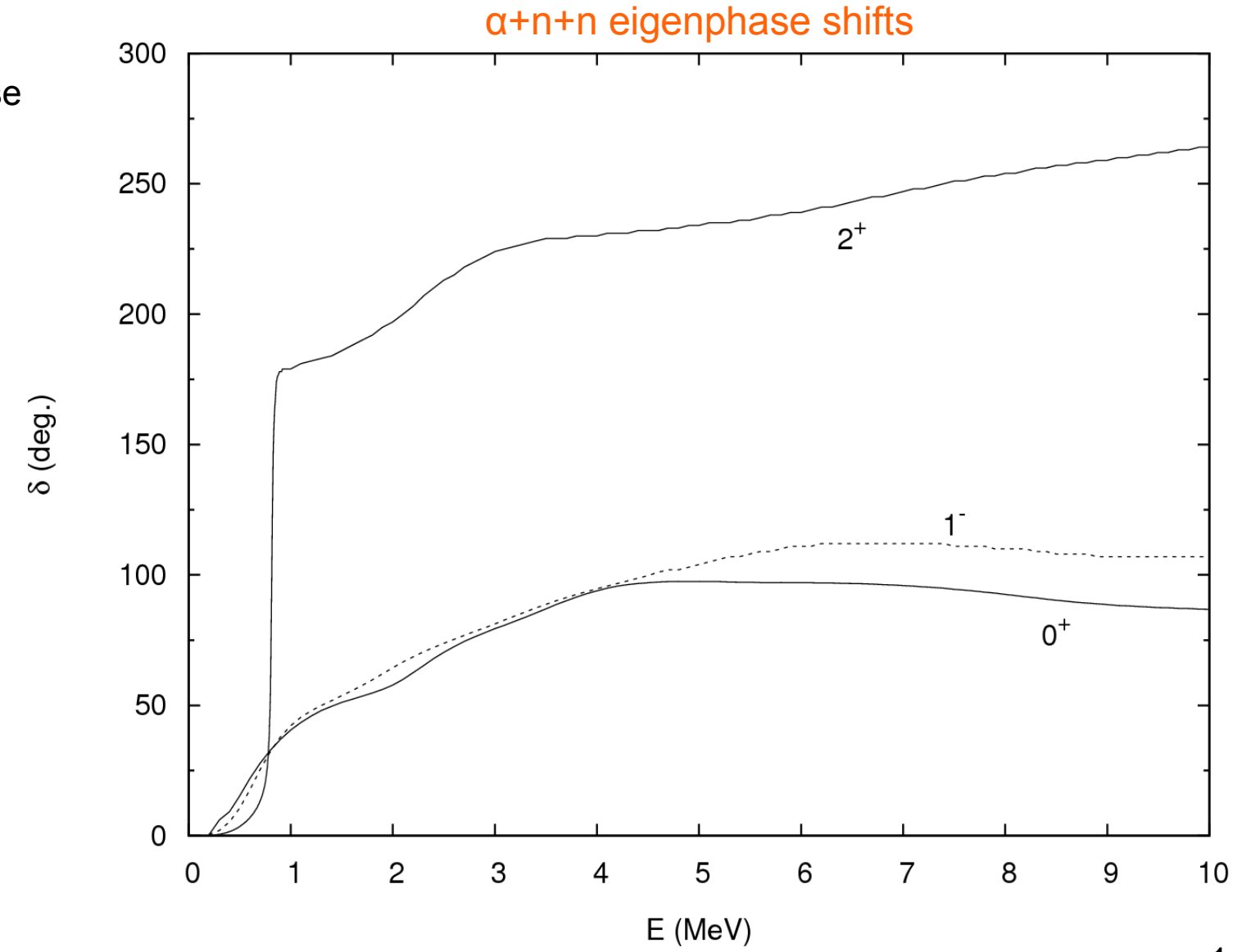
- Most important eigenphase shifts for different values of the channel radius a (R-matrix method parameter)
- Good stability under the change of a



^{6}He = α + n + n

Eigenphase shifts → $J^\pi = 0^+, 1^-, 2^+$

- Most important eigenphase shifts for different J^π
- 3-cluster structure for 0^+ and 1^-
- Narrow resonance at 0.8 MeV for $J^\pi=2^+$



Conclusions

- Microscopic description in the hyperspherical framework
- Bound states → Generator Coordinate Method.
  Scattering states → Generator Coordinate Method + R-matrix Method
- Development of a method in 3 steps (analytical, semi-analytical, numerical)
- Good results for ^{6}He :
  - → good convergence with K_{max}=24
  - → good stability with the channel radius a
  - → eigenphase shifts 0^{+} and 1^{-} shows an 3-cluster structure
  - → eigenphase shift 2^{+} shows a narrow resonance
- Outlook : Application to heavier nuclei such as ^{9}Be and ^{12}C

Generator Coordinate Method (GCM)

Calculation of hyperradial functions $\chi_\lambda^{J\pi}$? → variational method :

$$\chi_\lambda^{J\pi}(\rho) = \sum_n f_\lambda^{J\pi}(R_n)\; G_\lambda^{J\pi}(\rho, R_n) \qquad where \qquad \lambda = (K\ell_y\ell_x)$$

- $f_\lambda(R)$: generator functions
  - coefficients of the variational expansion → functions depending on a discretized variational coordinate R
  - R → generator coordinate
- G_λ : gaussian functions centered at R_n

Generator Coordinate Method (GCM)

In practice :

$$\Psi^{JM\pi} = \mathcal{A}\left[\phi_1\phi_2\phi_3 \sum_{\lambda}^{N_\lambda}\sum_{n}^{N_R} f_\lambda^{J\pi}(R_n)\; G_\lambda^{J\pi}(\rho, R_n)\; \mathcal{Y}_\lambda^{JM}(\Omega^5)\right] \quad (1)$$

$$= \sum_{\lambda}^{N_\lambda}\sum_{n}^{N_R} f_\lambda^{J\pi}(R_n)\; \Phi_\lambda^{JM\pi}(R_n) \quad (2)$$

equivalent

In theory : infinite → In practice : finite (N_λ ,N_R)

- $\Phi_\lambda(R_n)$: projected Slater determinants based on H.O. functions
  - constructed from a systematic analytical method (Mathematica, Mapple, ...)
  - basis size : N_λ x N_R
- $\Psi^{JM\pi}$ → Schrödinger equation $H\Psi^{JM\pi} = E^{J\pi}\Psi^{JM\pi}$ (numerical method) → bound states

Theoretical studies

- Bound-states : done ^{6}He and ^{6}Li → [S. Korennov, P. Descouvemont , Nucl. Phys. A 740 (2004) 249]